**Colorado Technical University**

 **Course:** MATH207 – Integral Calculus

#### Unit 7 Part 13 Readings: Convergence

**Convergence**

**Limit** of a series – does it converge on the right value or does it diverge?

**Squeeze theorem for sequences**

Let {an} {bn} and {cn}be sequences with an ≤ bn ≤ cn for all intergers n greater

than some index N

 lim

n →∞

 lim

k →∞

 lim

k →∞

If an = cn = L then bn = L

 lim

k →∞

**Divergence test:** If a series converges, then the = 0

 lim

k →∞

If ≠ 0, then the series diverges

**Harmonic series:** 1 + 1/2 + 1/3 + 1/4 + 1/5 + …

 Does it converge?

**Divergence test**

Cannot be used to prove convergence

Σ ak

 ∞

k=1

Form of series:

 lim

k →∞

Condition for divergence: (ak) ≠ 0

**Integral test:** Suppose *f* is a continuous, positive, decreasing function for x ≥ 1 and let

ak = ƒ(k) for k = 1, 2, 3, … then

Σ ark

 ∞

k=1

####  and

#### either both converge or both diverge

#### In the case of convergence, the value of the integral is *not*, in general, equal to

#### the value of the series.

**The p-series: t**he Integral test is used to analyze the convergence of an entire family of

 ∞

k=1

Σ

1

pk

infinite series known as the p-series:

**Ratio test:** If in a positive series the ratio of any general term to the preceding term

approaches a limit L as n →∞ then the series is convergent if L < 1 and

divergent if L> 1 or if the ratio →∞ as n →∞

If L = 1, the test fails

 ∞

k=1

Σ

1

kp

**Root Test:**

Let be an infinite series with nonnegative terms

 lim

k →∞

and let p = ak1/k

1. If 0≤p<1 the series converges

2. If p>1, (including p = ∞), the series diverges

****3. If p = 1, the test is inconclusive

#### Comparison Tests

 Used when all else fails

**Special Series and Convergence Tests**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Series or Test** | **Form of Series** | **Cond for Conv** | **Cond for Div** | **Comments** |
| **Geometric series** | Σ ark ∞k=1 | |r|<1 | |r|≥1 | If |r|<1 thensum = a/(1-r) |
| **Divergence test** | Σ ak ∞k=1 | Does not apply |  ak ≠ 0 limk →∞ | Cannot be used to prove conv |
| **Integral test** | Σ ak ∞k=1Where ak=ƒ(k) and ƒ is cont, pos and decr |   |  does not exist | The value of the integral is not the value of the series |
| **p-series** |  ∞k=1Σ 1pk  | p>1 | p≤1 | Useful for comparison tests |
| **Ratio test** | Σ ak ∞k=1where ak>0 |  limk →∞<1ak+1ak |  limk →∞>1ak+1ak | Inconclusive if limk →∞=1ak+1ak |
| **Root test** | Σ ak ∞k=1where ak≥0 |  < 1 limk →∞ |  > 1 limk →∞ | Inconclusive if  limk →∞ = 1 |
| **Comparison test** | Σ ak ∞k=1Σ bk ∞k=1where ak>0 | 0<ak≤bk and converges | 0<bk≤ak andΣ bk ∞k=1 diverges |  Σ ak ∞k=1 is given; you supply Σ bk ∞k=1 |
| **Limit comparison test** |  where Σ ak ∞k=1ak>0, bk>0 | 0≤ <∞Σ bk ∞k=1 limk →∞ andconverges |  > 0 limk →∞Σ bk ∞k=1 anddiverges |  Σ ak ∞k=1 is given; you supplyΣ bk ∞k=1 |
| **Alternating series test** |  Σ (–1)kak ∞k=1where ak>0, 0<ak+1≤ak |  ak = 0 limk →∞ |  ak ≠ 0 limk →∞ | Remainder Rn satisfies Rn < an+1 |