**Colorado Technical University**

**Course:** MATH207 – Integral Calculus

**Unit 3 Part 5 Readings: Improper Integrals**

**Improper Integrals**

One type of “improper” integral has ∞ or -∞ for one or both of

the limits

In general, replace the infinity with a variable (usually “b” or “a”),

do the integral and then take the limit of the result as“b”

or “a” goes to infinity

Integrals are “**convergent**” if the associated limit exists and is a

finite number (it’s not plus or minus infinity)

and “**divergent**” if the associated limit either doesn’t exist

or is (plus or minus) infinity

There are basically three cases:

1) *∫*a∞ƒ(x) dx is convergent

*∫*a∞ƒ(x) dx = lim(b→∞)*∫*abƒ(x) dx

2) *∫*-∞bƒ(x) dx is convergent

*∫*-∞bƒ(x) dx = lim(a→-∞)*∫*abƒ(x) dx

3) Both *∫*a∞ƒ(x) dx and *∫*-∞bƒ(x) dx are convergent

*∫*-∞∞ ƒ(x) dx = *∫*-∞b ƒ(x) dx + *∫*a∞ƒ(x) dx

= lim(b→∞)*∫*abƒ(x) dx + lim(a→-∞)*∫*abƒ(x) dx

Another type of improper integral has non-infinite limits BUT is

discontinuous in the interval

To solve these, split the area into “before the discontinuity” and   
 “after the discontinuity” and take the limits for each at the

discontinuity

Graphical user interface, application

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Diagram

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