**Colorado Technical University**

**Course:** MATH205 – Differential Calculus

#### Unit 2 Part 3 Readings: Modeling, and Transforming Functions

**Mathematical Modeling**

Functions are often used to model real-world phenomena

A **mathematical model** is a mathematical description (often by means of a function or

an equation) of a real-world phenomenon

The purpose of the model is to understand the phenomenon and often to make

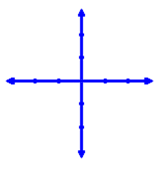
predictions about future behavior (**forecasts**)

A mathematical model is never a completely accurate representation of a physical

situation

Unless it was developed as part of a scientific experiment to show the particular

relationship, while it may be useful for forecasting, there is no scientific evidence



to show a particular model accurately explains a phenomenon

**Linear Functions**

Graphs are straight lines

Formula: *y* = ƒ(*x*) = *m*x + *b*

where m is the slope of the line and b is the y-intercept

Diagram

Description automatically generated

Constant Function

ƒ(*x*) = *c*

Chart

Description automatically generatedDiagram

Description automatically generated

Identity Function Absolute Value

ƒ(*x*) = *x* ƒ(*x*) = |*x|*

**Polynomial Functionss**

A function P is called a **polynomial** if

P(*x*) = *an xn* + *an*−1 *xn*−1 + ⋯ + *a*2 *x*2 + *a*1 *x* +​ *a*0

where *n* is a nonnegative integer

and the numbers *a*0*, a*1, *a***2**,…, *an* are constants called the **coefficients** of the polynomial

**Rational Functions**

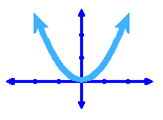
A **rational function** is a ratio of two polynomials:

ƒ(*x*) = P(*x*)/Q(*x*)

**Power Functions**

A function of the form ƒ(x) = xa, where a is a constant, is called a **power function**

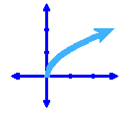
A fractional value of a will be a **root function**



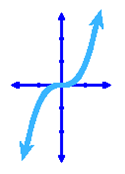
Square

or

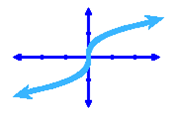
Quadratic



Square Root



Cube



Cube Root

**Trigonometric Functions**

For models, usually sine and cosine

These are used for real-life phenomena that are cyclical

**Exponential Functions**

Exponential functions are of the form *ƒ*(x) = ax, where the base a is a positive constant

Logarithmic functions *ƒ*(x) = logax, where the base a is a positive constant, are the

inverse functions of the exponential functions

**Transforming FunctionsDiagram

Description automatically generated**

vertical shifts

*y* = ƒ(*x*) + c plus shifts up

*y* = ƒ(*x*) – c minus shifts down

vertical stretching & shrinking

if c>1 *y* = c ƒ(*x*)

stretches vertically

if 0<c<1 *y* = c ƒ(*x*)

Diagram

Description automatically generatedshrinks vertically

horizontal shifts

*y* = ƒ(*x*)

*y* = ƒ(*x* – c)

Diagram

Description automatically generatedminus shifts right (counter-intuitive)

horizontal stretching & shrinking

if c>1 *y* = ƒ(c*x*)

shrinks horizontally

if 0<c<1 *y* = ƒ(c*x*)

stretches horizontally

Diagram

Description automatically generatedreflection

*y* = -ƒ(*x*)

reflects about the *x*-axis

*y* = ƒ(-*x*)

reflects about the *y*-axis

**Inverse Functions**

Inverse functions "undo" another function

The inverse is usually shown by putting a little "-1" after the function name: ƒ -1(*x*)

