**Colorado Technical University**

 **Course:** MATH116 – Foundations for Calculus

**Unit 3 Part 06 Readings: Polynomials**

**Algebra:** math formulas with unknown values represented by letters or symbols

**Algebra is symbolic:**

 **Constants**

 1 7.5 –42.631 π a b c i n

Sometimes constants are unknown.

They are given values like "i, j, k" if they are integers and "a, b, c" if they are rational numbers

 **Variables**

 x y z

 **Operators**

+ – × ÷ √ 2

 **Relationships**

 **= ≈** ≠ < >≤ ≥

**Polynomials** – algebraic expressions with only non-negative integer exponents

Note: An expression is not a polynomial if there is a variable in the denominator

"poly" means "many" "nomials" means "names" (variables)

7x2 – 3 x + 45 – 8xy + 601x2y – 400x2y2 + 85.1xy2 + 372.8y – 72y2

**Polynomial expressions contain:**

**variables** (x, y, z, ... )

**constants** (1, 3.4, ⅓, π, –2, ... )

**operators** (+, –, ×, ÷, 2, ... )

**Polynomial equations**

Contain an = sign

Mean ***exactly*** equal to

 Expression: a math formula with no "="

 If there is an "=", it is an equation, not an expression

**Parts of a polynomial:**

 **Terms** are separated by **+** – or  **=** signs

 The **variable term** in each term are all the variable symbols multiplied or divided in that

 term: the variable term in the term "34x2y" is "x2y"

 The number multiplying a variable term is called a **coefficient**

The number that is not multiplying any variable term is called the **constant**

**Degree of a polynomial**

 Sum the exponents in each term

 The largest sum is the degree of the polynomial

**Order of a polynomial**

A polynomial is usually arranged in descending order according to the degree of its

terms

 This is called "standard order" or "standard form"

**Evaluating** an algebraic expression – plug in the values given and solve using:

**Most of what you will use algebra for is evaluating mathematical models**

**PEMDAS !**

Order of operations

What calculation to do first:

Parentheses

Exponents (powers & roots)

Done at the same time left to right

>

Multiplication

Division

Done at the same time left to right

>

Addition

Subtraction

**Operations with Polynomials**

To combine two terms in a polynomial using addition or subtraction, they must be "like" terms. That is, it is necessary for them to have the same *base* and the same *exponent.*

**Adding Polynomials**

 (2*x* 3+ 4*x* + 3) + (5*x* 3+ 6*x* 2+ *x* + 7)

First, let's rewrite the addition to be vertical. Aligning "like" terms in columns:

2*x* 3+ +4*x* + 3

 + 5*x* 3+ 6*x* 2+ *x* + 7

 *--------------------------------------------*

Then, add the columns: 2*x* 3+ +4*x* + 3

 + 5*x* 3+ 6*x* 2+ *x* + 7

 *---------------------------------------------*

So your answer is: 7*x* 3 + 6*x* 2 +5*x* +10

**Subtracting Polynomials**

(2*x* 3+ 4*x* + 3)  (5*x* 3+ 6*x* 2+ *x* - 7)

Remember that the negative means you have to subtract everything inside the

 second set of parentheses, not just the first term

First, "negativize" everything inside the second set of parentheses:

–1  ( 5*x* 3+ 6*x* 2+ *x* + 7) = 5x3  6x2 x + 7

Now that the signs on the terms are correct, we are again ready to combine "like"

 terms:

First, let's rewrite the addition to be vertical. Aligning "like" terms in columns:

2*x* 3+ +4*x* + 3

5*x* 3 6*x* 2 *x* + 7

 *--------------------------------------------*

So your answer is: 3*x* 3  6*x* 2 +3*x* +

**Multiplying Polynomials**

Each of the terms in the first polynomial must be multiplied by each of the terms in the second polynomial. The BOX method is safest because it is easy to miss one of the pairs.

To multiply: (2*x*2 + 5*x*) (4*x*3+ 5*x*2+ 3) make a rectangle with each of the terms on the edge:

|  |  |  |  |
| --- | --- | --- | --- |
|  | 4*x*3 | 5*x*2 | 3 |
| 2*x*2 |  |  |  |
| 5*x* |  |  |  |

Then multiply each column value by each row value:

|  |  |  |  |
| --- | --- | --- | --- |
|  | 4*x*3 | 5*x*2 | 3 |
| 2*x*2 | **8*x*5** | **10*x*4** | **6*x*2** |
| 5*x* | **20*x*4** | **25*x*3** | **15*x***  |

Add together the like terms (frequently these are diagonal to each other), so the

 answer is:

8*x*5 + 30*x*4+ 25*x*3+ 6*x*2+ 15*x*

(You can include blank boxes if you like so that the diagonals are always like terms)

**Raising a Polynomial to a Power**

Remember that an exponent on an expression tells us how many times we are to

 multiply that expression by itself. If we have a polynomial expression like:

(3*x*2 4*x*) 2

we take the expression (3*x*2 4*x*) and multiply it by itself:

(3*x*2 4*x*) 2= (3*x*2 4*x*) (3*x*2 4*x*)

Using the BOX:

|  |  |  |
| --- | --- | --- |
|  | 3*x*2 | 4*x* |
| 3*x*2 | **9*x* 4** | **-12*x* 3** |
|  4*x* | **-12*x* 3** | **16*x* 2** |

So, the answer is: 9*x* 4 - 24*x* 3 + 16*x* 2

**Note: we will do ratios of polynomials later**

**Factoring Polynomials**

In this section, we reverse the techniques of polynomial multiplication

(*x* + 2)(*x*  2) = *x*2 4

Instead of multiplying two polynomials together to form a new polynomial:

we start with the solution and determine what two polynomials must be multiplied together to get it:

*x*2 4 = (*x* + 2)(*x*  2)

This method of breaking a polynomial such as *x*2 4 into a product of terms is called *factoring.*

Factoring is simply figuring out what expressions you would need to ***multiply*** together to get the expression you are given

**Factoring Technique 1 - Finding Common Factors**

##### Example: Factor the expression:x*2*y *+* 5xy

##### First factor each term in the expression:

 The factors of *x*2*y* are *x* *x* *y*

 The factors of 5*y*  are 5  *x*  *y*

 The factors the two terms have in common are *x* and *y*

Pulling out the common factors:

 ( *xy* ) (*x* + 5) are the factors of x*2*y *+ 5*xy

Checking: (*x* + 5)  ( *xy* ) = *x*(*x y*)+ 5 (*xy*) = *x*2*y* + 5*xy* , the original expression

**Factoring Technique 2 - The Difference of Two Squares**

The next-simplest method of factoring involves situations in which the expression is the difference of two expressions each of which is a perfect square.

*x 2*  4

*y* 2 25

*x 2*  16

*t* 2 9

Notice that in each case the first and second term is each a perfect square.

The factors of differences between squares follow this pattern:

*x 2*  4 = (*x* + 2)(*x*  2)

*y* 2 25 = (*y* + 5)(*y* 5)

The recipe for factoring this type of expression becomes even more apparent if we rewrite the left-hand side of both equations:

*x 2*  2 2 = (*x* + 2)(*x*  2)

*y* 2 5 2 = (*y* + 5)(*y* 5)

##### Example: Factor the following difference of two squares: 9x2 – 100

First, factor each term: the factors of 9*x* 2 are 3  3  *x*  *x* ,in other words, (3*x*) 2

 The factors of 100 are 2  2  5  5 , in other words, 10 2

So (3*x* + 10) (3*x* 10) are the factors of 9*x*2 100

**Factoring Technique 3 – Grouping Similar terms**

For expressions with **four** terms, sometimes you can factor them by grouping them into

two sets of two terms each

If the two sets have a common factor, it is easier to factor the full expression

In this class, the assignment will tell you if this technique should be tried, and the

grouping will always be the first two and the second two

##### Example: Factor the expression by grouping: x3 + 4x2 + 3x + 12

x3 + 4x2 + 3x + 12 = (x3 + 4x2) + (3x + 12)

 = x2(x + 4) + 3(x + 4)

 = (x2 + 3)(x + 4)

**Mathematical Models**

A mathematical model is a formula that gives results similar to those observed in a

real-life situation

In science and technology we often use mathematical formulas to calculate known

relationships

Many of these relationships are simple and well-known:

Distance = Rate × Time

Area rectangle = *L* × *W*

Volume cube = *L* × *W* × *H*

Area circle = *π* *r* 2

Circumference circle = 2*π r*

Ohm's Law: *V* = *I R*

****e = mc2

e = mc2 tells how much energy

 "e" can be generated

 using a mass "m"

When an atom of mass "m" is

 split, the amount of

 energy that is generated

 has been shown to be

 exactly what Einstein

 said it would be:

 m × c × c or mc2

Mathematical models are

 generally *estimates*;

 Einstein's is a *TRUE*

 equation proven in

the laboratory

**For calculus:**

The most important thing for

 calculus is that the

 functions must be

 smooth and

continuous

 No cliffs

 No holes



