**Colorado Technical University**

**Course:** MATH366 – Probability and Statistics

#### Unit 7 Part 14 Readings: Assignment

A combinatorial optimization problem assigning people to jobs, tasks to machines, etc.

The objective is to assign a number of resources to an equal number of activities so as to

minimize total cost or maximize total profit of allocation

Linear programming is used to solve these problems

An additional problem: you can’t assign half a person to a task or 0.93 of a machine, so you

may have to tweak the problem to find the optimal whole number

In Excel, set up a table with the employees/machines/whatever to be assigned on one axis

and the tasks on the other axis

|  |  |  |  |
| --- | --- | --- | --- |
| Worker: \ Task: | 1 | 2 | 3 |
| A |  |  |  |
| B |  |  |  |
| C |  |  |  |
| D |  |  |  |

Suppose you know the tasks and your workers well enough to know that some assignments

would cost less/take less time/produce fewer errors than others:

|  |  |  |  |
| --- | --- | --- | --- |
| Worker: \ Task: | 1 | 2 | 3 |
| A | 2.2 errors | 3.2 errors | 4.0 errors |
| B | 3.1 errors | 3.0 errors | 3.2 errors |
| C | 3.6 errors | 2.9 errors | 3.6 errors |
| D | 2.8 errors | 3.3 errors | 2.6 errors |

Obviously, for this problem, you would want Worker A to do task 1, Worker C to do task 2 and Worker D to do task 3.

But sometimes the solution is not obvious

**How many ways can you assign the projects to the employees?**

Project 1 could be assigned to any of your workers (n)

Project 2 could be assigned to any of your other workers (n-1)

Project 3 could be assigned to any of your other workers (n-2)

… and so on

The formula for how many ways you can assign “k” tasks to “n” employees (with no double assignments) is:

n! / (n-k)!

If the n employees can be assigned to multiple tasks, the formula for how many ways you can assign “k” tasks to “n” employees is:

nk

Suppose we have 3 employees to do 3 tasks

Suppose we know the average number of errors each employee makes on each task:

|  |  |  |  |
| --- | --- | --- | --- |
| Worker: \ Task: | 1 | 2 | 3 |
| A | 2.2 errors | 4.2 errors | 5.1 errors |
| B | 2.1 errors | 4.1 errors | 5.2 errors |
| C | 2.0 errors | 3.9 errors | 4.9 errors |

Suppose we DON’T want Worker C to do everything! What would be the best assignment to get the fewest errors assigning a different worker to each task?

Now you can create a new optimization table with k +1 = 4 columns and

and n! / (n-k)! +2 = 8 rows:

|  |  |  |  |
| --- | --- | --- | --- |
| Assignment | | |  |
| Task 1 | Task 2 | Task 3 | Expected Errors |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Fill in the possible worker to task assignments:

|  |  |  |  |
| --- | --- | --- | --- |
| Assignment | | |  |
| Task 1 | Task 2 | Task 3 | Expected Errors |
| A | B | C |  |
| A | C | B |  |
| B | A | C |  |
| B | C | A |  |
| C | A | B |  |
| C | B | A |  |

Fill in the expected errors for each assignment using the background data:

|  |  |  |  |
| --- | --- | --- | --- |
| Worker: \ Task: | 1 | 2 | 3 |
| A | 2.2 errors | 4.2 errors | 5.1 errors |
| B | 2.1 errors | 4.1 errors | 5.2 errors |
| C | 2.0 errors | 3.9 errors | 4.9 errors |

|  |  |  |  |
| --- | --- | --- | --- |
| Assignment | | |  |
| Task 1 | Task 2 | Task 3 | Expected Errors |
| A | B | C | 2.2 + 4.1 + 4.9 |
| A | C | B | 2.2 + 3.9 + 5.2 |
| B | A | C | 2.1 + 4.2 + 4.9 |
| B | C | A | 2.1 + 3.9 + 5.1 |
| C | A | B | 2.0 + 4.2 + 5.2 |
| C | B | A | 2.0 + 4.1 + 5.1 |

Calculate the expected number of errors for each assignment (actually, in Excel the calculations are done automatically):

|  |  |  |  |
| --- | --- | --- | --- |
| Assignment | | |  |
| Task 1 | Task 2 | Task 3 | Expected Errors |
| A | B | C | 2.2 + 4.1 + 4.9 = 11.2 |
| A | C | B | 2.2 + 3.9 + 5.2 = 11.3 |
| B | A | C | 2.1 + 4.2 + 4.9 = 11.2 |
| B | C | A | 2.1 + 3.9 + 5.1 = 11.1 |
| C | A | B | 2.0 + 4.2 + 5.2 = 11.4 |
| C | B | A | 2.0 + 4.1 + 5.1 = 11.2 |

Which assignment should you use? The one that would be expected to have the fewest number of errors: Worker B gets Task 1, Worker gets Task 2, and Worker A gets Task 3.

**Probability Assignments**

Some Assignment problems are about time

Some are about cost or profit

Some are about probabilities - we will have to multiply the probabilities to get the total

probability

Suppose we have 2 pitchers on our baseball team: A and B

Historically, when we play our 3 opponent teams (1, 2 and 3), the probability of winning

when each pitcher was pitching was:

|  |  |  |  |
| --- | --- | --- | --- |
| Pitcher \ Team | 1 | 2 | 3 |
| A | 54% | 49% | 50% |
| B | 51% | 49% | 52% |

What is the assignment of pitchers to the games that will ensure the highest probability of

winning?

It may seem obvious to use Pitcher A for Team 1 and Pitcher B for Team 3, but what about

Team 2?

First we need the calculate the number of possible assignments:

For this type of problem, you may want to use the same pitcher for all three teams (or

not)

There are two pitchers possible for each team: 2 x 2 x 2 = 6 assignments

|  |  |  |  |
| --- | --- | --- | --- |
| Assignment | | |  |
| Team 1 | Team 2 | Team 3 | Probability of Winning |
| A | A | A | .54 \* .49 \* .50 = .1323 |
| A | B | A | .54 \* .49 \* .50 = .1323 |
| A | B | B | .54 \* .49 \* .52 = .137592 |
| B | B | B | .51 \* .49 \* .52 = .129948 |
| B | B | A | .51 \* .49 \* .50 = .12495 |
| B | A | B | .51 \* .49 \* .52 = .129948 |

Which assignment will give you the highest chance of winning?

Have Pitcher A against Team 1 while pitcher B goes against Teams 2 and 3