**Colorado Technical University**

 **Course:** MATH366 – Probability and Statistics

#### Unit 7 Part 13 Readings: Types of Discrete Probability Distributions

#### Monte Carlo Simulations:

 includes a random input

 rely on repeated random sampling to obtain numerical results

 use random inputs to solve problems that might be deterministic (non-random)

 Often used for financial planning:

 https://www.portfoliovisualizer.com/monte-carlo-simulation#analysisResults

 https://retirementplans.vanguard.com/VGApp/pe/pubeducation/calculators/

RetirementNestEggCalc.jsf

 https://www.retirementsimulation.com

#### Types of Discrete Probability Distributions:

#### Uniform - all outcomes are equally likely

 **Binomial** - two possible outcomes (with replacement)

 **Hypergeometric** - two possible outcomes (without replacement)

 **Poisson** - arrivals

**Binomial Probability**

Some events have only two solutions: yes/no right/wrong male/female

head/tail success/failure

These events are called “binomial” (two names)

Requirements:

A fixed number of repeated trials

The number of trials is called "n"

The trials are independent (the outcome of one trial will not affect the

outcome of another trial)

Each trial has only two possible outcomes - a success or a failure

You must define what constitutes a "success"

The number of successes is called "X"

The outcomes are mutually exclusive

The probability that a particular outcome will occur on any given trial is

constant

The probability of one outcome is "p"

The probability of the other outcome is "q"

q = 1 – p (for p expressed as a decimal)

The probability of X successes out of n binomial trials:

P = nCx px q n-x

Use: https://stattrek.com/online-calculator/binomial.aspx or Excel BINOM.DIST(X,n,p,FALSE)

The mean of the binomial distribution is: np

The variance is: npq

 **Monte Carlo simulation for binomial:**

https://shiny.rit.albany.edu/stat/binomial/

**Negative Binomial Probability Distribution**

 a discrete probability distribution concerning the number of trials that must occur in

order to have a predetermined number of successes

 Assume Bernoulli trials:

1) there are two possible outcomes

2) the trials are independent

3)  p, the probability of success, remains the same from trial to trial

 Let X denote the number of trials until the rth success

The probability function of X is:

P(X=x) = (x−1)C(r−1)(1−p)x−rpr

for x=r,r+1,r+2,…

 In Excel: =NEGBINOM.DIST(#failures,#successes,prob success, true/false)

 Or: https://stattrek.com/online-calculator/negative-binomial.aspx

 The expected value of X in a negative binomial distribution with parameters (r, p) is:

μX = r/p

 The variance of X in a negative binomial distribution with parameters (r, p) is:

σX2 = r(1-p)/p2

 Subtypes:

The geometric distribution is a special case of the negative binomial

distribution

It deals with the number of trials required for a single success

 The Pascal distribution for the case of an integer-valued stopping-time

parameter r

The Polya **distribution for the real-valued case**

**Hypergeometric Probability Distribution**

The binomial without replacement

Requirements:

There is a finite known population size N

There are exactly K objects with a desired feature in the population

A fixed number of objects are drawn

The number drawn is called "n"

The trials are *dependent* because of non-replacement (the outcome of one

trial will affect the outcome of another trial)

Each trial has only two possible outcomes - the chosen object has a given feature or

does not have it

The number chosen having the feature is called "k"

The probability of k objects with the feature out of n selected is:

P = $\frac{ }{}$

Use: http://stattrek.com/online-calculator/hypergeometric.aspx

 or Excel HYPGEOM.DIST(k,n,K,N,FALSE)

The mean of a hypergeometric distribution is: n (K/N)

The variance is: n(K/N)((N-K)/N)((N-n)/(N-1))

**Monte Carlo simulation for hypergeometric**:

https://keisan.casio.com/exec/system/1180573202

**Poisson Probability Distribution**

**Poisson** - P(# arrivals) exceeds a certain number

Requirements:

 A given number of events

 Designated "k"

 A fixed interval (time or space)

 Events happen with a known constant rate

The average number of events in an interval is designated λ (lambda)

Events happen at a time or space independent of the last event's time or

space

The probability of observing k events in an interval is:

P = e −λ $\frac{λ^{k}}{k!}$

Use: http://stattrek.com/online-calculator/poisson.aspx

 or Excel POISSON.DIST(k,λ,FALSE)

The mean of a Poisson Distribution is: λ

The variance is: λ

**Monte Carlo simulation for Poisson**:

https://keisan.casio.com/exec/system/1180573180