**Colorado Technical University**

**Course:** MATH207 – Integral Calculus

#### Unit 7 Part 13 Readings: Convergence

**Convergence**

**Limit** of a series – does it converge on the right value or does it diverge?

**Squeeze theorem for sequences**

Let {an} {bn} and {cn}be sequences with an ≤ bn ≤ cn for all intergers n greater

than some index N

lim

n →∞

lim

k →∞

lim

k →∞

If an = cn = L then bn = L

lim

k →∞

**Divergence test:** If a series converges, then the = 0

lim

k →∞

If ≠ 0, then the series diverges

**Harmonic series:** 1 + 1/2 + 1/3 + 1/4 + 1/5 + …

Does it converge?

**Divergence test**

Cannot be used to prove convergence

Σ ak

∞

k=1

Form of series:

lim

k →∞

Condition for divergence: (ak) ≠ 0

**Integral test:** Suppose *f* is a continuous, positive, decreasing function for x ≥ 1 and let

ak = ƒ(k) for k = 1, 2, 3, … then

Σ ark

∞

k=1

#### and

#### either both converge or both diverge

#### In the case of convergence, the value of the integral is *not*, in general, equal to

#### the value of the series.

**The p-series: t**he Integral test is used to analyze the convergence of an entire family of

∞

k=1

Σ

1

pk

infinite series known as the p-series:

**Ratio test:** If in a positive series the ratio of any general term to the preceding term

approaches a limit L as n →∞ then the series is convergent if L < 1 and

divergent if L> 1 or if the ratio →∞ as n →∞

If L = 1, the test fails

∞

k=1

Σ

1

kp

**Root Test:**

Let be an infinite series with nonnegative terms

lim

k →∞

and let p = ak1/k

1. If 0≤p<1 the series converges

2. If p>1, (including p = ∞), the series diverges

**Diagram, text

Description automatically generated**3. If p = 1, the test is inconclusive

#### Comparison Tests

Used when all else fails

**Special Series and Convergence Tests**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Series or Test** | **Form of Series** | **Cond for Conv** | **Cond for Div** | **Comments** |
| **Geometric series** | Σ ark  ∞  k=1 | |r|<1 | |r|≥1 | If |r|<1 then  sum = a/(1-r) |
| **Divergence test** | Σ ak  ∞  k=1 | Does not apply | ak ≠ 0  lim  k →∞ | Cannot be used to prove conv |
| **Integral test** | Σ ak  ∞  k=1  Where ak=ƒ(k) and ƒ is cont, pos and decr |  | does not exist | The value of the integral is not the value of the series |
| **p-series** | ∞  k=1  Σ  1  pk | p>1 | p≤1 | Useful for comparison tests |
| **Ratio test** | Σ ak  ∞  k=1  where ak>0 | lim  k →∞  <1  ak+1  ak | lim  k →∞  >1  ak+1  ak | Inconclusive if  lim  k →∞  =1  ak+1  ak |
| **Root test** | Σ ak  ∞  k=1  where ak≥0 | < 1  lim  k →∞ | > 1  lim  k →∞ | Inconclusive if  lim  k →∞  = 1 |
| **Comparison test** | Σ ak  ∞  k=1  Σ bk  ∞  k=1  where ak>0 | 0<ak≤bk and  converges | 0<bk≤ak and  Σ bk  ∞  k=1  diverges | Σ ak  ∞  k=1  is given;  you supply  Σ bk  ∞  k=1 |
| **Limit comparison test** | where  Σ ak  ∞  k=1  ak>0, bk>0 | 0≤ <∞  Σ bk  ∞  k=1  lim  k →∞  and  converges | > 0  lim  k →∞  Σ bk  ∞  k=1  and  diverges | Σ ak  ∞  k=1  is given;  you supply  Σ bk  ∞  k=1 |
| **Alternating series test** | Σ (–1)kak  ∞  k=1  where  ak>0, 0<ak+1≤ak | ak = 0  lim  k →∞ | ak ≠ 0  lim  k →∞ | Remainder Rn satisfies  Rn < an+1 |